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A PROCEDURE FOR DESIGNING EXACT RECONSTRUCTION FILTER  
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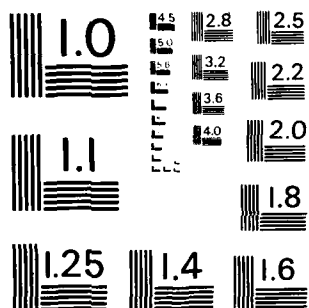
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A PROCEDURE FOR DESIGNING EXACT RECONSTRUCTION FILTER BANKS FOR TREE-STRUCTURED SUBBAND CODERS

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ABSTRACT

In recent years, tree-structured analysis/reconstruction systems have been extensively studied for use in subband coders for speech. In such systems, it is important that the individual channel signals be decimated in such a way that the number of samples coded and transmitted does not exceed the number of samples in the original speech signal. Under this constraint, the systems presented in the past have sought to remove the aliasing distortion while minimizing the overall analysis/reconstruction distortion. In this paper, it is shown that it is possible to design tree-structured analysis/reconstruction systems which meet the sampling rate condition and which also result in exact reconstruction of the input signal. This paper develops the conditions for exact reconstruction and presents a general method for designing the corresponding high quality analysis and reconstruction filters.

INTRODUCTION

In a recent paper by Barnwell [1], it was shown that alias-free reconstruction using recursive and non-recursive filters was possible where the analysis/reconstruction section had no frequency distortion or no phase distortion, but not both. In the development that follows, the coefficient symmetry condition on the analysis filters is lifted and exact reconstruction free of aliasing, phase distortion and frequency distortion is shown to be possible using FIR filters. In addition, the filter constraints that enable perfect reconstruction are discussed and an easily implementable design procedure providing high quality filters is presented.

Exact Reconstruction

The frequency division of the subband coder is performed in the analysis stage as shown in Figure 1, where  $H_0(e^{j\omega})$  and  $H_1(e^{j\omega})$  are lowpass and highpass filters respectively. To preserve the system sampling rate, both channels are decimated resulting in the two down-sampled signals,  $Y_0(e^{j\omega})$  and  $Y_1(e^{j\omega})$ . In the reconstruction section, the bands are recombined by up-sampling and filtering to give the reconstructed signal

$$\hat{x}(e^{j\omega}) = \frac{1}{2} [H_0(e^{j\omega})G_0(e^{j\omega}) + H_1(e^{j\omega})G_1(e^{j\omega})] X(e^{j\omega}) \quad (1)$$

$$+ \frac{1}{2} [H_0(-e^{j\omega})G_0(e^{j\omega}) - H_1(-e^{j\omega})G_1(e^{j\omega})] X(-e^{j\omega})$$

The frequency response of the 2-band system is represented by the first term in equation (1), while the second term is the aliasing.

To obtain exact reconstruction, consider the case in which the analysis/reconstruction filters are designed using the assignments

$$G_0(e^{j\omega}) = H_0(-e^{j\omega}) \quad (2a)$$

$$G_1(e^{j\omega}) = H_0(-e^{j\omega}) \quad (2b)$$

$$H_1(e^{j\omega}) = H_0(-e^{j\omega}) \quad (2c)$$

where  $H_0(e^{j\omega})$  is not required to be linear phase.

This assignment eliminates the aliasing term in equation (1) and the resulting overall system function is given by

$$\begin{aligned} C(e^{j\omega}) &= \frac{1}{2} H_0(e^{j\omega}) H_0(-e^{j\omega}) + \frac{1}{2} H_0(-e^{j\omega}) H_0(-e^{j\omega}) \\ &= \frac{1}{2} F_0(e^{j\omega}) + \frac{1}{2} F_1(e^{j\omega}) \end{aligned} \quad (3)$$

where  $F_0(e^{j\omega})$  and  $F_1(e^{j\omega})$  will be called the "product filters." It is clear from equation (3) that

$$F_1(e^{j\omega}) = F_0(-e^{j\omega})$$

Now assume that the product filters are zero phase FIR filters. Then the exact reconstruction condition is given by

$$c(n) = \mathcal{F}^{-1}\{C(e^{j\omega})\} = \delta(n) \quad (4)$$

where  $\mathcal{F}^{-1}\{\cdot\}$  denotes the inverse Fourier transform. The corresponding condition on the product filter is given by

$$f_0(n) \left[ \frac{1+(-1)^n}{2} \right] = \delta(n) \quad (5)$$

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So in order for exact reconstruction to be obtained, there are two conditions on the product filters which must be met. First, they must meet the condition of equation (5). Second, they must be decomposable into analysis and reconstruction filters in such a way that equation (3) is valid.

In order to see how these conditions can be met, it is appropriate to decompose the product filters into their zero time and nonzero time components, giving

$$F_0(e^{j\omega}) = V(e^{j\omega}) + A \quad (6)$$

in the frequency domain or, equivalently,

$$f_0(n) = v(n) + A\delta(n) \quad (7)$$

in the time domain. In these expressions,  $v(n)$  is constrained to have no zero time component, i.e.,  $v(0) = 0$ , and the exact reconstruction condition of equation (10) is met so long as  $A = 1$  and

$$v(2n) = 0 \quad n = 0, \pm 1, \pm 2, \dots \quad (8)$$

or, equivalently

$$v(e^{j\omega}) = -v(-e^{j\omega}) \quad (9)$$

This is the class of all filters which are anti-symmetric about the Nyquist frequency, as illustrated in Figure 3. This constraint can be met in several ways and a number of powerful filter design tools are available to design the required filters.

#### Filter Design

In the design procedure, the approach is to first design the product filter,  $F_0(z)$ , and then to decompose  $F_0(z)$  into  $H_0(z)$  and  $H_0(z^{-1})$ . There are a number of techniques which can be used to design  $F_0(z)$ .

For example, any filter of the form

$$f(n) = v(n) \frac{\sin \frac{\pi n}{2}}{\pi n} \quad (10)$$

where  $v(n)$  is a window function, will satisfy the exact reconstruction condition of equation (5). Similarly, optimal equiripple product filters can be constructed using a Remez exchange algorithm. In such designs (in which the Chebyshev error is minimized with equal weighting over the entire frequency band), only those solutions with the largest number of extrema, i.e. the extraripple filters, will satisfy equations (8) and (9). Both the Parks-McClellan and the Hofstetter algorithms for equiripple filter design may be used to design the product filters.

The attenuation of the analysis filter and product filter are related in the following way

$$A_p = 2A_a + 20 \log_{10} (2 \cdot 10^{-A_a/10}) \quad (11)$$

where  $A_p$  is the product filter attenuation and  $A_a$  is the analysis filter attenuation. The second term in this expression is due to the difference in the transition width of  $F_0(z)$  and  $H_0(z)$ . This tends to make the Hofstetter algorithm [2] more attractive for the equal ripple designs since the attenuation is an explicit input to the design procedure. It is important to notice that the transition width of the analysis filter will be larger than that of the product filter and therefore care should be exercised in using the Parks-McClellan algorithm.

The second condition which must be met is that the product filters must be factorable in such a way that equation (2c) is valid. The additional constraint that equation (2c) places on  $F_0(z)$  can be stated as follows: for every zero of  $F_0(z)$  at  $re^{j\theta}$ , there must be another corresponding zero at  $(1/r)e^{j\theta}$ . If this condition is met, then  $F_0(z)$  can always be written as the product of reciprocally paired zeros

$$F_0(z) = G \prod_{m=1}^M (z - z_m)(z^{-1} - z_m^*) \quad (12)$$

where  $G$  is a real constant. An important point to note is that any FIR filter which is both real and whose coefficients are symmetric in the time domain (zero phase) comes close to meeting this condition. In particular, the zeros of any such real, symmetric FIR filter must either meet the above condition, lie in complex zero pairs on the unit circle or occur at  $z = \pm 1$ . These two conditions become identical if there is a further requirement that any zero on the unit circle must be a double zero. The important point here is that any product filter which meets the exact reconstruction condition of equation (10) can be easily transformed into a new product filter which also meets the factorization condition required to satisfy equation (2c). The required transformation is simply

$$f_0(n) \rightarrow af_0(n) + b \quad (13)$$

where "a" and "b" are real constants.

This is exactly the strategy outlined by Herrmann and Schussler [3] for designing minimum phase FIR filters. The starting point is a symmetric, zero phase half-band filter with an odd number of coefficients. Figure 3 shows a real and symmetric product filter (equation 13) with "b" equal to zero. Picture now, if you will, the same filter with "b" equal to a half. The stopband ripples now occur above and below the real axis. These zero crossings mark the location of roots on the unit circle. As "b" is increased, the symmetry conditions are not changed, but the zeros on the unit circle migrate toward one another and leave the unit circle in pairs. When "b" is large enough, all of the zeros will have either been driven off the unit circle or will have become double zeros on the unit circle (Figure 2). For equiripple filters, it is possible to have many double zeros on the

unit circle, while for window designs there will generally only be one double zero pair. The effect of the multiplier constant "a" is to rescale the impulse response to have the correct passband gain. It can be easily shown that

$$a=1+10^{-A_p/20} \quad (14a)$$

$$b=10^{-A_p/20} \quad (14b)$$

Once the appropriate product filters have been designed, it is straightforward to extract the analysis and reconstruction filters. For this case, the product filter can be characterized as in equation (12) and, therefore

$$P_1(z) = P_0(-z) = G \prod_{n=1}^M (z+z_n)(z^{-1}+z_n^{-1}) \quad (15)$$

where G is a constant. The corresponding analysis and reconstruction filters are given by

$$H_0(z) = \sqrt{G} \prod_{n=1}^M (z-z_n) \quad (16a)$$

$$H_1(z) = \sqrt{G} \prod_{n=1}^M (z^{-1}+z_n^{-1}) \quad (16b)$$

$$G_0(z) = \sqrt{G} \prod_{n=1}^M (z^{-1}-z_n^{-1}) \quad (16c)$$

$$G_1(z) = \sqrt{G} \prod_{n=1}^M (z+z_n) \quad (16d)$$

Filters of this type give the desired exact reconstruction.

#### Discussion

A close examination of equations (16a-16d) reveals some interesting points. First, for each zero pair in  $P_0(z)$  (one at  $re^{j\theta}$  and one at  $(1/r)e^{-j\theta}$ ) one of the two zeros is always included in the analysis filter while the other is included in the reconstruction filter. Hence, the analysis filters and the reconstruction filters always have identical magnitude responses, and this magnitude response is exactly the square root of  $P(e^{j\omega})$ . Therefore,  $P(e^{j\omega})$  must be designed to be the square of the desired analysis filter magnitude.

Second, note that for a given product filter, there are many choices of  $H_0(z)$  that have a magnitude response equal to the square root of the product filter magnitude. Among these is the minimum phase analysis filter consisting of roots strictly inside the unit circle and one root from each double zero pair on the unit circle. The

remaining roots comprise the corresponding reconstruction filter which, in this case, would have maximum phase. Figs. 4 and 5 show the normalized magnitude response and group delay respectively of a 32 coefficient minimum phase analysis filter. Also shown (Fig. 6) is an approximate linear phase analysis filter obtained by selecting conjugate pairs of product filter roots that occur alternately inside and outside of the unit circle as the frequency increases.

Third, note that the analysis filters are related in that  $h_1(n)$  is formed by reversing  $h_0(n)$  in time and multiplying by  $(-1)^n$ . The implication is that the filter coefficient symmetry present in QMFs is absent here and therefore the polyphase structure [1,4] cannot be used in implementation.

The major point is that exact reconstruction analysis and reconstruction filters are not difficult to design. In addition, the exact reconstruction filters generated by this technique generally have better characteristics in terms of their transition widths and attenuation than the most published QMFs of the same length, and, of course, they also give exact reconstruction.

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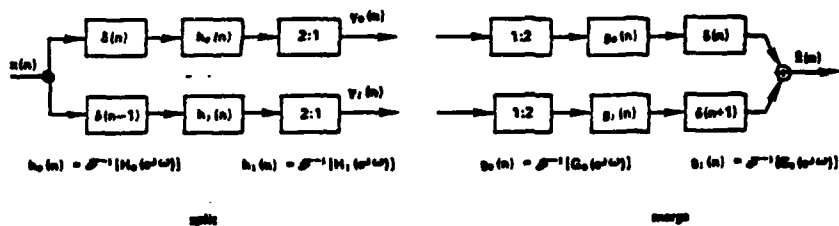


FIGURE 1. 2-BAND SUBBAND CODER.

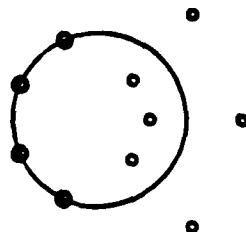


FIGURE 2. POLE/ZERO PLOT OF PRODUCT FILTER.

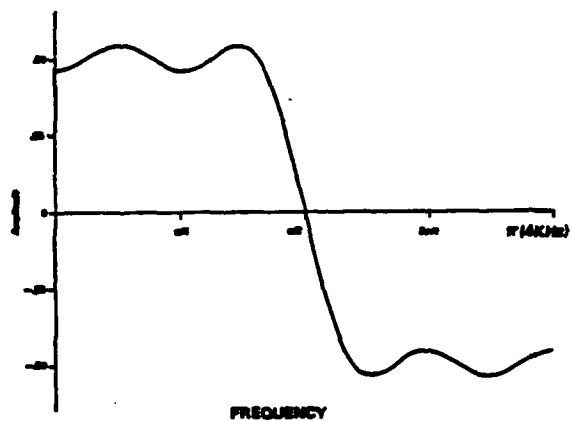


FIGURE 3. NONZERO TIME COMPONENT OF PRODUCT FILTER.  $V(e^{j\omega})$ .

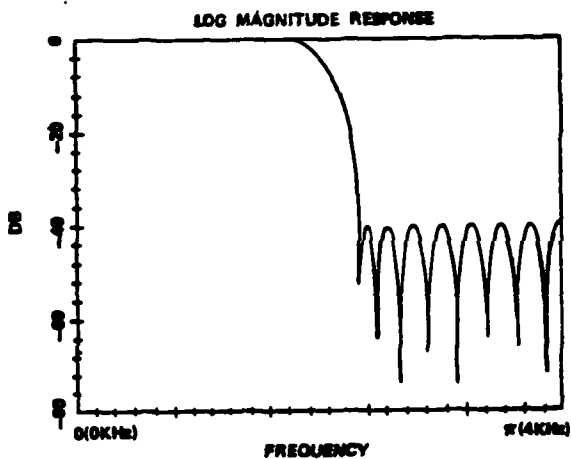


FIGURE 4. 32-TAP MINIMUM PHASE FILTER.

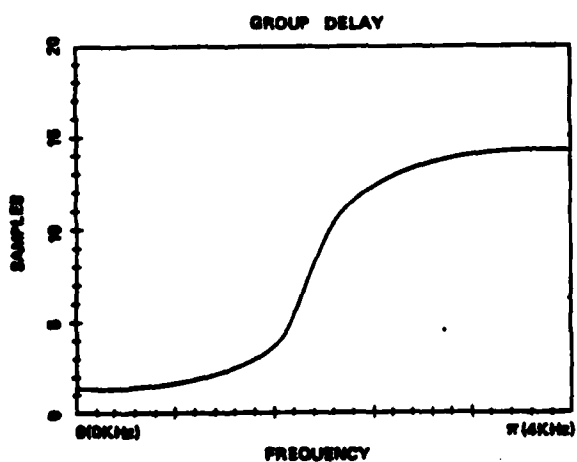


FIGURE 5. 32-TAP MINIMUM PHASE FILTER.

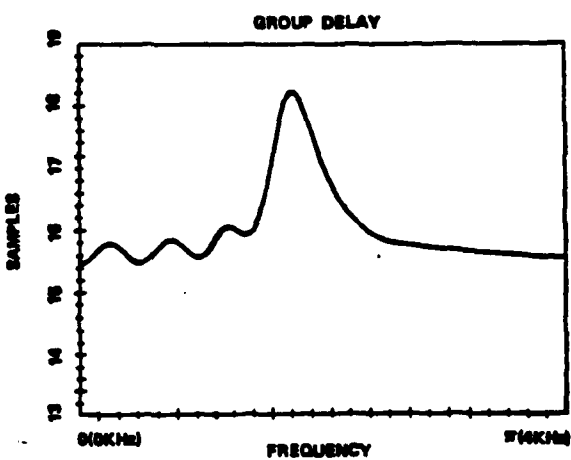


FIGURE 6. 32-TAP APPROXIMATELY LINEAR PHASE FILTER.

